

Multiple Stable Hydrological States in Models: Implications for Water Resource Management

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EXTENDED ABSTRACT

Many models of surface and groundwater hydrology are constructed without thought given to the possibility of multiple stable states for the same parameter set. If a single stable state is assumed, a transient hydrological disturbance of any magnitude will result in a return of the model to the same stable state, and thus show an infinite resilience. In an attempt to make this assumption less implicit, a plausible numerical hydrological model is presented in which this assumption is violated.

The model is developed from the assumption that multiple coexistent hydrological states do not exist. Its aim is not to prove such multiple equilibria exist but rather demonstrate that minor, defensible and plausible changes to groundwater-vadose zone interaction equations give rise to multiple equilibria.

The impetus for the model derives from the concept of ecosystem resilience. It is based upon the hillslope Boussinesq partial differential equation (PDE) coupled to a vertically integrated unsaturated zone ordinary differential equation (ODE) and solved numerically. The only significant change from traditional coupled models is the reduction in leaf area index (LAI) and thus transpiration as the watertable, which is assumed highly saline, approaches the surface. This is demonstrated to produce a positive feedback and two coexistent equilibrium watertable depths.

If a watertable has only one equilibria, then irrespective of the initial head it will eventually come to the same equilibrium. To investigate the potential for two equilibria the model was solved with an initial head of 5% and 92.5% of the maximum saturated thickness. Figure 1 show results for a saturated lateral hydraulic conductivity, k_{sat} , of 0.16 m/day. Clearly the model has both a deep and shallow watertable equilibria. To highlight that all dryland catchments are unlikely to have two such equilibria, the simulation is repeated for a high and low k_{sat} . For the low k_{sat} the system has only a shallow equilibria, while for the high k_{sat} only deep equilibria.

The water resource impacts from climate change may be significantly amplified for catchments having two equilibria. For a catchment currently at the shallow equilibria, more extended periods of reduced infiltration may increase the probability of a shift to the deep equilibria. Upon a shift, streams hydrologically connected to the aquifer may shift from gaining to losing. The stream baseflow would reduce, suffer ecologically and, regionally, yields would further decline under climate change. If one attractor instead existed, changes would likely be more proportional to the change in recharge and thus less significant. While somewhat alarmist, and the model developed thus far omits the interaction with streams, it highlights the change in natural resource outcomes assuming two attractors as opposed to one.

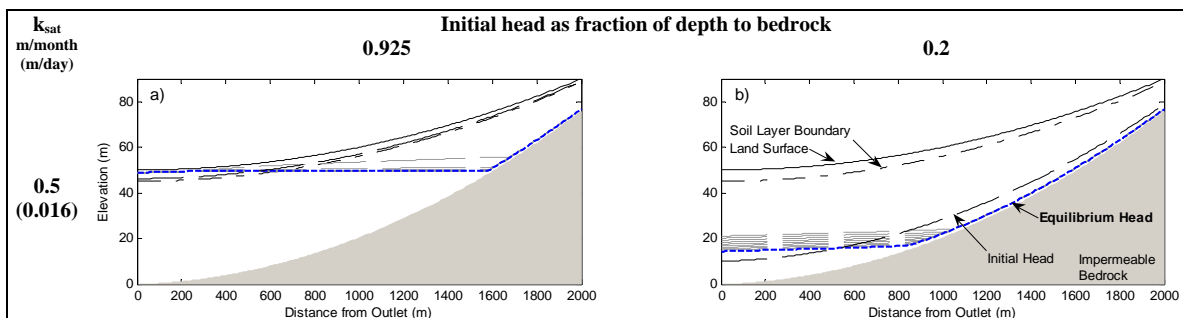


Figure 1. Solution cross sections of watertable elevation at (---) initial condition; (—) 40 year time steps; and (-.-) at equilibrium.

INTRODUCTION

Many models of surface and groundwater hydrology are constructed without thought given to the possibility of multiple stable states for the same parameter set. If a single stable state is assumed, a transient hydrological disturbance of any magnitude will result in a return of the model to the same stable state, and thus show an infinite resilience. In an attempt to make this assumption less implicit, a numerical hydrological model is presented in which this assumption is violated.

The impetus for the model derives from the concept of ecosystem resilience (referred to hence forth as resilience) (Walker et al. 2004). It is a concept, derived from dynamic systems theory, in which biophysical environments are considered to have multiple possible equilibria (hence forth *attractors*) for the same *parameters* and within each one, the system stochastically moves. For a sufficient transient disturbance the system may cross a threshold (hence forth *repeller*) and move into an alternate, and possible socially undesirable, equilibrium. The only quantitative catchment-resilience model is that of Anderies (2005) and extended by Peterson et al. (2005). This model, which is an annual time step, salt and water, groundwater-unsaturated zone lumped model of the Goulburn catchment (Victoria, Australia), predicts it to have two coexistent equilibria. As a result of widespread land clearing it predicts one of the equilibria to have been lost and the catchment to have only the state of near-zero depth to groundwater remaining.

While the concepts of resilience and multiple equilibria (i.e. an attractor) are worthy of inquiry, neither calibration nor validation of such models is almost ever undertaken. While it is questionable whether an unobserved attractor can be numerically validated via data from the observed attractor, validation of the model to the observed attractor is thought a necessary first step. This investigation develops a resilience model built from parameters independently observable and of a structure enabling future calibration. As per Anderies (2005), the focus is vadose-hydrogeological interactions and the model is distributed to facilitate calibration to observed groundwater levels.

MODEL DEVELOPMENT

The model is developed from the assumption that multiple coexistent hydrological states do not exist. Its aim is not to prove such attractors exist but rather that minor, defensible and plausible changes to groundwater-vadose zone interaction equations give rise to multiple attractors.

For model transparency and tractable calibration the model is developed with minimal state variables and parameters. While soil and aquifer salinity are potential drivers of multiple states, these state variables are thus omitted as data are generally unavailable though required for initial conditions and desirable for calibration. The model state variables are water table elevation and soil moisture. The model is constructed to be continuous and smooth, i.e. without thresholds from min/max, if/else functions. This has the advantage of reducing the stiffness of the differential equations, which enhances the validity of the solution; allows future numerical continuation of attractors; and the future use of gradient based calibration techniques. As a compromise between the high frequency soil moisture dynamics and the low frequency lateral groundwater flow, the model time step is monthly.

Unsaturated Zone Model

The vadose zone-watertable literature predominately emerges from Australian salinity management investigations. For irrigated lucerne with a one metre deep watertable it was found that with a change from fresh (0.1 dS m⁻¹) to saline (16 dS m⁻¹) water table, the leaf area index declined by 41%; transpiration by 36%; and groundwater uptake by 67% (Zhang et al. 1999). Uptake from the saline ground water was estimated at 3.3 and 2.3 mm day⁻¹ of which 19 percent (0.21 and 0.15 mm day⁻¹ respectively) was estimated to be transpired.

Summarising twenty field studies, Thorburn (1997) concludes that, following establishment of the plant, groundwater uptake of shallow saline groundwater by trees and pastures is very comparable to that of bare soil. A final study of 80 sites found that groundwater uptake is minor for sites with saline groundwater and observed water level declines are likely to be due to initial direct uptake followed by reduced in-situ recharge (George et al. 1999).

With respect to field studies of tree water use, an in-situ investigation of *Eucalyptus largiflorens* trees found that at sites recharged only by vertical infiltration, uptake was from fresh unsaturated deep drainage rather than accessible saline ground water (Holland et al. 2006). Conversely, Thorburn et al. (1993) concluded groundwater uptake was an important source but acknowledged that a high saline watertable may have restricted transpiration and uptake. Returning to plant growth, the leaf area per tree of *E. camaldulensis* and *E. occidentalis* after seven years growth was found to decrease by 50% and 61% respectively at sites of moderately saline shallow groundwater compared to fresh sites (Benyon et al. 1999).

Although these studies differ in location, and thus climate, geology, soil etc; species; method and aims, it is plausible to summarise that i) transpiration declines as a shallow saline watertable approaches the surface; and ii) the reduction in transpiration is coupled with a reduction in LAI. In the following model development this forms the basis for the modified transpiration function.

In the development of the monthly time step model, the monthly change in soil moisture cannot be assumed zero and thus requires a soil moisture state variable(s). As this investigation is into long-term multiple equilibria of the water table rather than soil moisture dynamics, the soil moisture is vertically integrated to a single layer store. The surface slope is sufficiently flat that lateral unsaturated zone flux is also assumed zero. The unsaturated zone point water balance is thus:

$$\frac{dm(t)}{dt} = P(t) - I(S,t) - Q(m,S,t) - T(m,S,t) - E(m,S,t) - L(m,S,t) - U(m,S) \quad (1)$$

where $m(t)$ [$L T^{-1}$] is soil moisture; $P(t)$ is monthly precipitation; $I(S,t)$ is canopy interception rate and S is groundwater storage; $Q(m,S,t)$ is runoff rate; $T(m,S,t)$ is transpiration rate; $E(m,S,t)$ is soil evaporation rate; $L(m,S,t)$ is leakage to, or uptake from, the watertable; and $U(m,S)$ is uptake of soil moisture with a rising watertable. The development of each flux term is beyond the scope of this paper.

The transpiration flux is central to this investigation into multiple equilibria. Transpiration is frequently modelled as a piecewise linear function of soil moisture fraction (e.g. Laio et al. 2001). Above a relative soil moisture threshold, evapotranspiration is at climatic demand. Long term simulations do though require a change in the maximum transpiration potential to reflect changes in vegetation growth and seasonality. A simple such model for a water limited environment is that proposed by Johnson and Thornley (1983), in which the transpiration potential is a function of both LAI and monthly PET, multiplied by the relative soil moisture fraction:

$$T = PET \left(1 - e^{-k_{light} LAI} \right) \frac{\theta - \theta_{wp}}{\theta_* - \theta_{wp}} \quad (2)$$

where, T [$L T^{-1}$] is the transpiration rate at time t ; PET [$L T^{-1}$] the areal potential evapotranspiration at time t ; k_{light} the canopy light extraction coefficient; LAI the leaf area index; θ the soil moisture fraction constrained between θ_{wp} and θ_* ; θ_{wp} the wilting point soil moisture fraction; and θ_* the soil moisture fraction at which stomata begin to close. Tuteja et al. (2004) estimated monthly LAI as the

product of the monthly average LAI by the ratio of the month's precipitation to the month's average precipitation. The above discussion of the impact of a shallow saline water table on transpiration indicates that transpiration declines with both increased groundwater salinity and rising watertable and that LAI is a plausible indicator of this decline. As the above transpiration potential is limited only by climate it is inadequate for the investigation of transpiration in saline conditions. A simple modification is to make LAI also a function of standing water level (SWL) such that as the watertable lowers from the surface to far below the root zone the LAI increases from zero to the climatically limited maximum. Using a two parameter polynomial logistic function, LAI is thus estimated as:

$$LAI = LAI_i \left(\frac{P}{P_i} \right)^\beta \left(\frac{d_v^\alpha}{d_v^\alpha + d_{LAI/2}^\alpha} \right) \quad (3)$$

where, LAI_i the fixed average LAI for month i ; P [L] the precipitation for time t , P_i [L] the average monthly precipitation for month i ; and β a scaling parameter for change in LAI with the precipitation ratio; d_v [L] is the vertical depth to the watertable; $d_{LAI/2}$ [L] a parameter for the watertable depth at which LAI is 50% of LAI_i ; and α a parameter for the rate of decline of LAI with d_v .

Modification of Hill-Slope Boussinesq Model

The inclusion into the above model of lateral flow allows for the consideration of heterogeneous potentiometric curvature in the determination of multiple water table equilibria. It also allows the future calibration to distributed observation bores. The hillslope storage Boussinesq (HSB) model, a one dimensional lateral flow model accounting for catchment shape and the slope of the impermeable bed, was selected (Troch et al. 2003). As the HSB catchment geometry (catchment width, slope and depth to basement) is defined by few model parameters, it also facilitates tractable investigation into their role in multiple attractors emerging.

Application of the HSB model to dryland salinity problems is likely to misrepresent dryland salinity processes due to its constant bed-slope angle. As illustrated by the common example of a shallow watertable occurring at the break of slope, topographic curvature heavily influences the depth to watertable and cannot be assumed zero for this investigation. Hilberts et al. (2004) proposed a non-uniform sloping bed HSB such that the slope angle, i , is a function of distance to the outlet, x . This expansion of the Troch et al. (2003) Boussinesq model provided the basis for the necessary expansions for a soil layer of differing porosity to

the underlying consolidated sediments; recharge as a function of depth to watertable; and uptake (deposition) of soil moisture when the watertable rises within the soil layer. Development of this HSB model is beyond the scope of the paper, though the HSB equation is:

$$\frac{dS}{dt} = \frac{\frac{kS}{\mu} \left(\sin i \frac{\partial i}{\partial x} \frac{\partial b}{\partial x} + \cos i \frac{\partial^2 b}{\partial x^2} + \cos i \frac{\partial i}{\partial x} + \left(\frac{1}{k} \frac{\partial k}{\partial x} + \frac{1}{S} \frac{\partial S}{\partial x} - \frac{1}{\mu} \frac{\partial \mu}{\partial x} \right) \left(\cos i \frac{\partial b}{\partial x} + \sin i \right) \right) + w(L-E)}{1 - \frac{\theta}{\Phi}} \quad (4)$$

where b is the saturated thickness [L]; i the slope angle; E [L] groundwater evaporation; k [L/T] the lateral saturated hydraulic conductivity; L [L/T] the leakage to the watertable from the unsaturated zone; S [L²] the groundwater storage; w [L] the catchment width at x ; μ the aggregate aquifer porosity allowing a change in the porosity as the watertable enters the soil layer; θ the soil moisture fraction; Φ the soil porosity; and σ a dimensionless vector smoothing the soil-aquifer boundary.

Numerical Solution

Analytical solutions to the HSB PDE have been derived with a uniform bedrock slope angle and recharge. An analytical solution to the above non-linear coupled vadose-HSB PDE (eqn. 1 and 4) is both very unlikely and not the focus of this paper. Numerical solution methods were therefore adopted. They were solved numerically as a transient problem using a MatLab variable time step solver *ode15s* (Shampine and Reichelt 1997) with finite difference Jacobian. The partial derivatives for the groundwater storage, S , were estimated via finite difference whereby the first derivative was centre weighted. The discretisation of x was 10 metres and solved at a relative and absolute error tolerance of $1e^{-4}$ and $1e^{-6}$ respectively. Validation solution involved i) verification of all analytical differential equations against finite difference estimates; ii) confirmation of conservation of mass; and iii) validation of the mass balance errors being proportional to the spatial discretisation size and relative and absolute error.

MODEL EXPLORATION

In exploration of model behaviour, arbitrary parameters typical of a semi-arid catchment ($P/PET = 0.5$) are presented in Table 1. The model parameters for the baseline exploration (Table 1) were selected to ensure two states are produced while adhering to the model assumptions: the saturated conductivity is homogenous and isotropic; and plant water use is supply limited. The land use is grazed pastures and the Brooks and Corey soil parameter are for a clay loam (Rawls et al. 1982). The three equations in Table 1 simply produce

smooth sub-yearly sinusoidal estimates. The catchment width is also given in Figure 2.

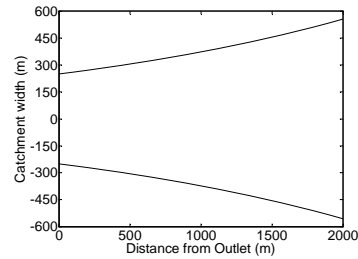


Figure 2. Catchment width

Table 1. Model Parameters

Hydrogeological Parameters:	
Lateral saturated hydraulic conductivity: k_{max}	5 m/month
Specific yield: f	0.05
Lower boundary condition – hydraulic gradient:	0.003
Upper boundary condition – specified flux	0 m ³ /mth
Unsaturated Zone Parameters:	
Maximum infiltration	100 mm/ month
Soil depth (as fraction of depth to bedrock)	0.1
Brooks and Corey soil parameters: ϕ_c, ψ_a	0.242, 0.5643m
Vertical conductivity: k_v	1680 mm/ month
Porosity: Φ	0.464
Residual soil moisture fraction: θ_r	0.075
Plant Water Use	
Stomata closure soil moisture: θ_s	0.35
Wilting point soil moisture (at -1.5 MPa): θ_{wpt}	0.1752
Canopy light extraction coefficient: K_{light}	0.6
Evaporation depth at 5% relative potential: d_{evap}	0.5 m
Depth at which LAI is 50% of potential: $d_{LAI/2}$	2 m
LAI rate of decay with watertable depth: a	5
Change in LAI with the rainfall ratio: β	1
Average monthly LAI = $LAI_{min} + \frac{LAI_{max} - LAI_{min}}{2} \left(1 - \cos \left(2\pi Mod \left(t - \frac{1}{12}, 1 \right) \right) \right)$	LAI _{min} = 0.8 LAI _{max} = 3.2
Climatic Data	
Monthly precipitation = $P_{min} + \frac{P_{max} - P_{min}}{2} \left(1 - \cos \left(2\pi Mod \left(t - \frac{1}{12}, 1 \right) \right) \right)$	P _{min} = 15 mm/ month P _{max} = 60 mm/ month
Monthly areal PET = $PET_{min} + \frac{PET_{max} - PET_{min}}{2} \left(1 - \cos \left(2\pi Mod \left(t - \frac{1}{12}, 1 \right) \right) \right)$	PET _{min} = 30 mm/mnth PET _{max} = 120mm/mnth

If a watertable has only one attractor then irrespective of the initial head or size of disturbance, it will eventually come to the same equilibrium. To investigate the potential for two attractors the model was solved with an initial head of 5% and 92.5% the maximum saturated thickness. Importantly though, it is implausible to expect all dryland catchments to have the same number of attractors. The model exploration thus investigates the response to the two initial heads at three values of saturated lateral conductivity, k_{sat} .

Figure 3a-f shows the simulations from the two initial heads and three values of k_{sat} . Displayed in each cross section is the initial head; the transient solution at 40 year increments; and the steady state estimate. Clearly, for a k_{sat} of 5 m/month (0.16 m/day) the solutions converge to two different attractors. For an initial (shallow) head of 92.5% of

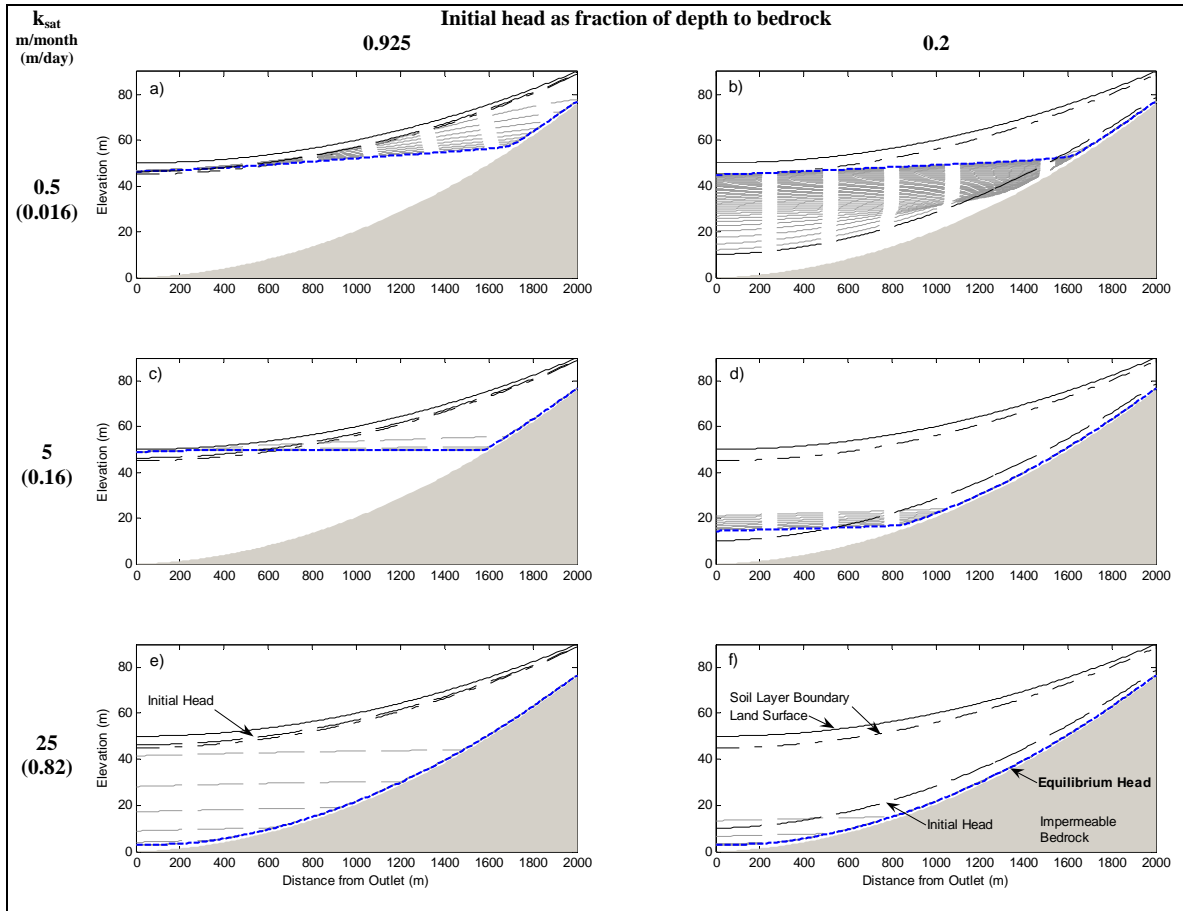


Figure 3. Solution cross sections of watertable elevation at (---) initial condition; (—) 40 year time steps; and (-.-.-) at equilibrium.

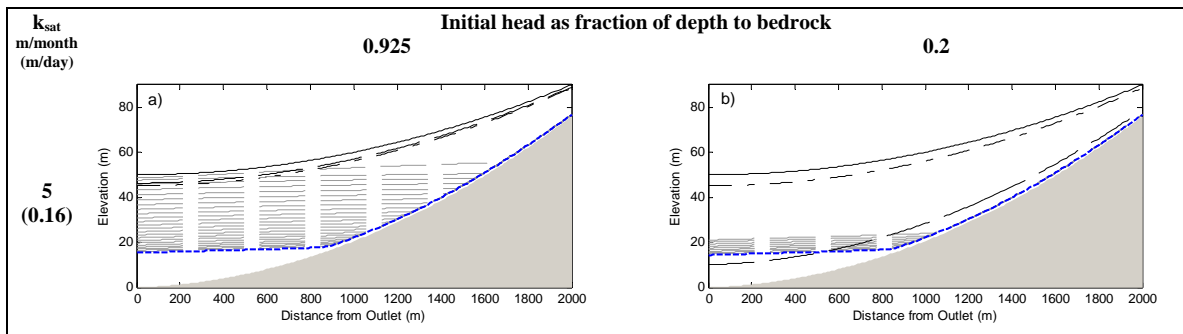


Figure 4. Solution cross sections (from model in which LAI is watertable independent) of watertable elevation at (---) initial condition; (—) 40 year time steps; and (-.-.-) equilibrium.

the depth to bedrock, the solution converges to an equilibrium watertable depth of 1.1 metres (0 m from the outlet). Conversely, for an initial (deep) head of 20% of the depth to bedrock, the solution converges to an equilibrium watertable depth of 36 metres (0 m from the outlet). Respectively, the model has a shallow and a deep attractor, and thus the equilibrium watertable depth is very dependent upon the initial head.

The number of attractors is, however, dependent upon k_{sat} . For a higher k_{sat} of 25 m/month (0.82 m/day), both the shallow and deep initial condition

solutions converge to the same attractor at a watertable depth of 47 metres (0 m from the outlet). While for a k_{sat} of 0.5 m/month (0.016 m/day) both the shallow and deep initial condition solutions converge to a shallow attractor at a watertable depth of 3.8 metres (0 m from the outlet).

To provide a comparison with more traditional groundwater models, and to make transparent the process causing the two attractors, the LAI (eq. 3) is made independent of the watertable depth via removal of the equation's logistic function. This is the only change in the model, data or parameters

from the model producing the results of Figure 3. Figure 4a-b shows that for a higher k_{sat} of 5 m/month (0.16 m/day), both the shallow and deep initial condition solutions converge to the same attractor at a SWL of 35.65 metres (at 0m from the outlet), which is identical to the deep state attractor of Figure 3d. Thus the inclusion into the model of LAI, and thus transpiration, as a function of SWL results in the two attractors of Figure 1c-d.

DISCUSSION AND CONCLUSION

This investigation has found that a simple coupled vadose-groundwater model can have two qualitatively different watertable equilibria. Central to the emergence of more than one attractor is the inclusion of a positive feedback, which in this model is the reduction in LAI as a shallow saline watertable approaches the surface. More specifically, when the saline watertable is near surface, a recharge event resulting in a watertable rise of dh_t causes the LAI to further decline which in turn causes the transpiration to further decline and thus a greater fraction of infiltration going to recharge.

Not all catchments are thought to have multiple equilibria. Their emergence is dependent upon the catchment hydrogeology, shape and recharge. Figure 3e-f demonstrate that for a high lateral saturated conductivity, the transmissivity at the shallow initial head is sufficient that flow out of the lower boundary is greater than the increased recharge from the positive feedback and thus only the deep attractor exists. Conversely, Figure 3a-b demonstrate that for a low lateral saturated conductivity, the transmissivity at the deep initial head is insufficient such that the lower boundary outflow is less than the recharge, causing only the shallow attractor to emerge. Between these two conditions, as demonstrated, both attractors can exist. Importantly, the values of saturated conductivity over which two attractors are observed is dependent upon the soil texture. More conductive soils resulted in more recharge and thus high minimum values of saturated conductivity at which both attractors were observed.

While the above conclusions are interesting, the coupled model on which it is based is very simplified. Most notable is the unsaturated zone ODE. The monthly time step results in a dampened within-month soil moisture fraction and, due to the very non-linear unsaturated vertical conductivity, lower recharge than is expected. The omission of explicit salt dynamics is probably the biggest limitation of the model. The model assumes the impacts of salt from a rising watertable on LAI are immediate and at a pseudo-equilibrium. While the sensitivity to salt is simulated via two parametric

parameters (Table 1: $d_{LAI/2}$ and α) the rate of salt flushing and its correlation with soil type is ignored. While these deficiencies may compromise the findings, the model is felt sufficiently valid to challenge the assumption of hydrological systems having only one attractor. The simplicity of the model also allows future tractable calibration and limit cycle continuation, which quantifies the resilience with a change in, say, k_{sat} . Both are unlikely if expanded to a coupled vertically distributed advection-dispersion – HSB model.

A possible criticism of the findings is that, if catchments have multiple attractors, why have they not been observed? To observe a change from one attractor to another and attribute this to their being two attractors would require long term groundwater head monitoring of a catchment that has not undergone notable landuse change; is of a configuration likely to have two attractors; and to have experienced a climatic disturbance sufficient to cause a change of attractors. Unfortunately this seems unlikely and understanding may need to rely on plausible numerical models. While difficult to prove, the potential consequences for assuming a catchment has one attractor when in fact it has two, or more, need discussion. At the local scale, dryland salinity mitigation could be enhanced via identification of catchments with two attractors and temporary interventions undertaken to shift the watertable over the repeller to the deep watertable attractor. Assuming all catchments have a single attractor instead requires permanent, and thus more costly, interventions. At the wider scale, and more topical, the water resource impacts from climate change may be significantly amplified for catchments having two attractors. For a catchment currently at the shallow attractor, more extended periods of reduced infiltration may increase the probability of a shift to the deep attractor. Upon a shift, streams hydrologically connected to the aquifer may shift from gaining to losing. The stream baseflow would decline, suffer ecologically and, regionally, yields would further decline under climate change. If one attractor existed instead, changes would likely be more proportional to the change in recharge and thus less significant. While somewhat alarmist, and the model thus far omits the interaction with streams, it highlights the change in natural resource outcomes assuming two attractors as opposed to one.

Future work will involve *limit cycle* continuation and calibration to dryland catchments of the Goulburn, Victoria. The presented positive feedback are unlikely to be the only instance within catchment hydrology. Further work will also explore and develop additional potential examples.

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